Mathematical Grammar School

a school with the High National Distinction status

Mathematical Grammar School Cup BELGRADE

The Mathematical Grammar School Cup

Physics Competition

24.06.2021.

Instructions:

1. *Duration of the competition is* 3h*. Maximum number of points is* 80*.*
2. *Use the answer sheet only to give answers to* ***all*** *the questions.*
3. *Use of calculators is allowed. If needed assume the gravitational acceleration to be* $10\ {}^{m}/{}_{s^2}$*.*
4. *It is not allowed to write on anything other than the answer sheet, notebook and the paper with questions.*
5. *When finished, turn in the answer sheet and the notebook.*

Good luck! ☺

Circle the correct answer

1. *(1 point)* After which scientist was the SI unit for electric resistance named:

(A) Isaac Newton **(B)** James Watt **(C)** Andrè-Marie Ampére **(D)** Alessandro Volta **(E)** George Ohm

2. *(1 point)* How can the unit for electric resistance be expressed in terms of other SI units?

(A) $J \cdot C$ **(B)** $\frac{V}{A}$ **(C)** $J \cdot A$ **(D)** $\frac{W}{A}$ **(E)** $\frac{W}{C}$

3. *(1 point)* What is the proper chronological order, from earliest to latest, of the important work done by these famous scientists in developing our understanding of electromagnetism?

(A) Faraday, Maxwell, Coulomb **(B)** Coulomb, Maxwell, Faraday
(C) Faraday, Coulomb, Maxwell **(D)** Maxwell, Faraday, Coulomb
(E) Coulomb, Faraday, Maxwell

4. *(1 point)* The photograph on the right shows a cup with a small coin at the bottom before and after the cup was filled with water. Which of the following phenomena explains why the coin is visible once we pour water in the cup (photo on the right labelled after)?

(A) diffraction of light **(B)** refraction
(C) interference of light **(D)** scattering of light
(E) total internal reflection

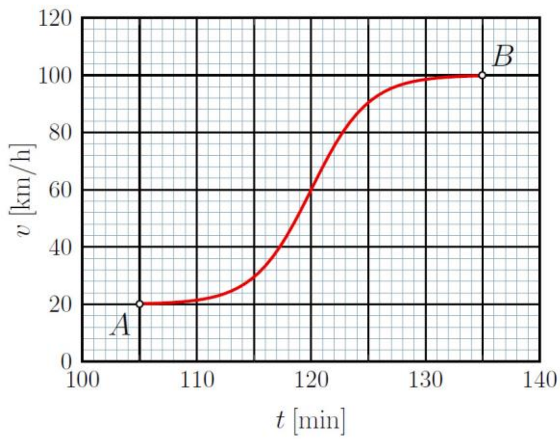
5. *(1 point)* Who was the first person to measure the speed of light?

(A) Galileo Galilei **(B)** Albert Michelson **(C)** Isaac Newton **(D)** Ole Roemer **(E)** Hippolyte Fizeau

6. *(1 point)* One of Europe's first suspension bridges, Broughton Suspension Bridge, was built over a river Irwell in Grater Manchester, England in 1826. On the 12th of April 1831 the bridge collapsed when a military troop marched over it in step, throwing about forty of the soldiers into the water or against the chains. All the soldiers survived to tell that they felt the bridge begin to vibrate in time with their footsteps. Which physical phenomenon led to the collapse of the bridge?

(A) resonance **(B)** weight of the soldiers **(C)** friction **(D)** a sudden gust of wind
(E) buoyancy

7. *(2 point)* A material particle is moving on a straight line in one direction by varying its velocity v with time t, as recorded in the graph. The particle leaves station A at the moment $t_A = 105$ min with an instantaneous velocity $v_A = 20$ km/h, whereas later on it arrives at station B at the moment $t_B = 135$ min with instantaneous velocity $v_B = 100$ km/h. Bounded with A and B, $v(t)$ curve possesses central symmetry with respect to the graph point (120 min, 60 km/h). Find the distance between the two stations, $\overline{AB}$, as well as the maximum acceleration of the particle, a_{max}, attained over its travelling from A to B.

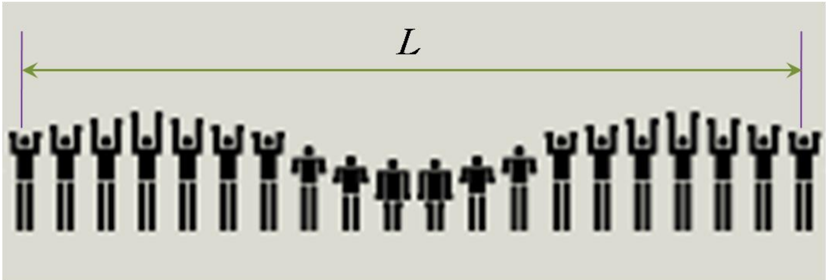


(A) $\overline{AB} = 30$ km and $a_{max} \approx 1/81$ m/s^2 **(B)** $\overline{AB} = 20$ km and $a_{max} \approx 1/81$ m/s^2

(**C**) $\overline{AB} = 30$ km and $a_{max} \approx 1/27$ m/s^2 (**D**) $\overline{AB} = 40$ km and $a_{max} \approx 1/9$ m/s^2

(**E**) $\overline{AB} = 20$ km and $a_{max} \approx 1/27$ m/s^2

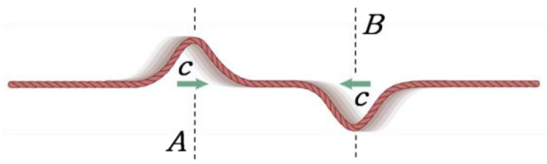
8. *(2 point)* During an inaugural event such as the MG Cup 2021 opening ceremony, a number of the Mathematical Grammar School pupils send a human wave in front of a spectator while snapshots were being taken. As the wave-front reaches a pupil, she/he reacts standing up with a cheer to ultimately sit within a regular time interval. The pupils proceed cooperatively in such a manner that each of them causes a constant delay of $\tau = 110$ ms in the predecessor's reaction. There are $n = 20$ students over the total length of $L = 10.45$ m, as shown in one of the snapshots. What speed c of such a human-wave propagation will the spectator observe?



(**A**) $c = 98.0$ m/s (**B**) $c = 5.00$ m/s (**C**) $c = 102$ m/s (**D**) $c = 4.75$ m/s

(**E**) $c = 100$ m/s

9. *(1 point)* Propagating along a taut rope, two oppositely deformed pulses whose centres stand initially $|AB| = 8$ cm apart, are moving towards each other, as shown in the figure. The speed of each pulse is $c = 2$ cm/s. After $t = 2$ s, the total mechanical energy of the pulses will be:



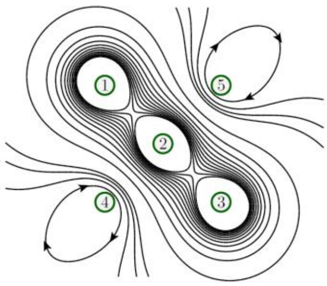
(**A**) zero (**B**) purely potential (**C**) purely kinetic (**D**) partly kinetic and partly potential

(**E**) negative

10. *(1 point)* A solid metal ball immersed in alcohol would weigh Q_0 at 0°C, but Q at a higher temperature. It is known that, with an increasing temperature, the metal thermally expands slower than alcohol. If the mass density of the metal is large compared to that of alcohol, one can demonstrate that:

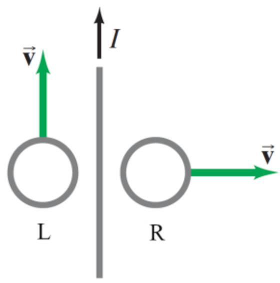
(**A**) $Q_0 > Q$ (**B**) $Q_0 = Q$ (**C**) $Q_0 < Q$ (**D**) all of these (**E**) none of these

11. *(2 point)* The figure shows a cross section of five long parallel wires carrying equal currents directly into ($\otimes$) or out of the page ($\odot$), four of them arranged at the corners of the square (denoted as $3, 4, 5,$ and 1), whereas the remaining one passes through its centre (denoted as 2). The configuration of their magnetic induction lines surrounding them is also visualized. Respecting the ascending order of the numeral wire notation, find the correct order of their current senses.



(**A**) $\otimes\otimes\otimes\odot\odot$ (**B**) $\otimes\odot\otimes\odot\odot$ (**C**) $\odot\otimes\odot\otimes\otimes$ (**D**) $\odot\otimes\odot\odot\odot$
(**E**) $\odot\odot\odot\otimes\otimes$

12. *(2 point)* Two loops of wire (L and R) are moving in the vicinity of a very long straight wire carrying a steady current. See the figure where the two loops span the same plane with the wire. Find the correct answer for the direction of the induced current in each loop.



(**A**) zero for L and counter-clockwise for R

(**B**) counter-clockwise for L and clockwise for R

(**C**) counter-clockwise for L and zero for R (**D**) zero for L and clockwise for R

(**E**) clockwise for L and counter-clockwise for R

13. *(1 point)* When you look at the image of the Moon from a rippling sea, it appears elongated (left photo) as compared to a "normal" image when the water surface is perfectly flat and still acting as a mirror (right photo). Explain why?

(**A**) There is a vertical gradient in the profile of the wind velocity that generates local temperature fluctuations which influence the moonlight propagation - a phenomenon highly akin to mirages emerging in deserts.

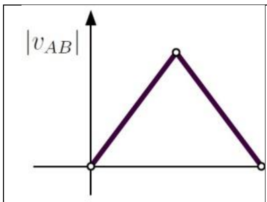
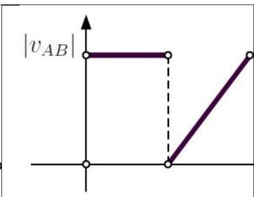
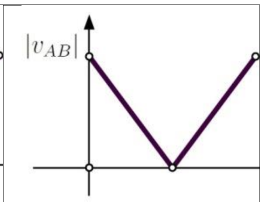
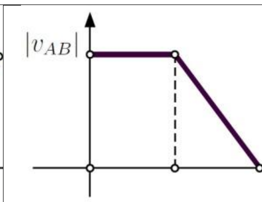
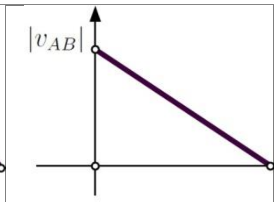
(**B**) The ripples on the water surface result in random incident reflection angles falling on the observer's eye across the distance of the water's surface - an illusion of a continuously elongated image is created.

(**C**) The left photography portrays the marine water, while the freshwater surface is given on the right - the very origin of the effect lies in the salinity.

(**D**) The wind sweeping through the air above the water draws the moonlight towards the observer to make it appear elongated.

(**E**) The phenomenon is purely cosmic and can be properly explained only if the Moon's position relative to the observer is specified.

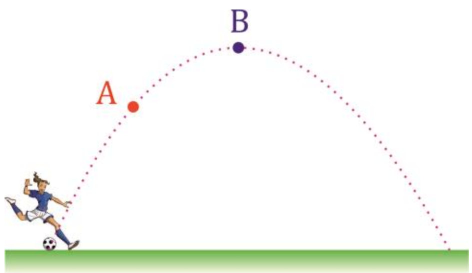
14. *(2 point)* Body A is thrown vertically upwards with such a velocity that it reaches a maximum height of h. Simultaneously, another body B is dropped from height h. It strikes the ground and does not collide with A nor rebounds. The module of velocity of A relative to B versus time graph, $|v_{AB}(t)|$, is best represented by which of these?

				
(A)	**(B)**	**(C)**	**(D)**	**(E)**

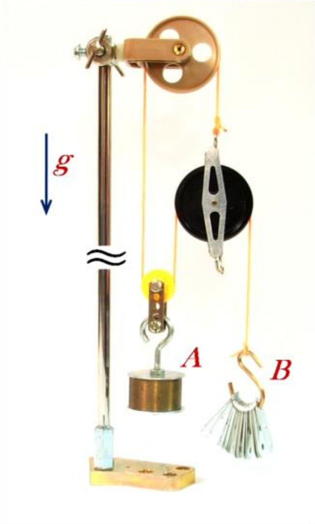
15. *(1 point)* Soccer player kicks a football that consequently follows a parabolic trajectory. Which statement about the magnitude of acceleration of the ball at points A and B is the correct one?

(A) $A > B$ **(B)** $A < B$ **(C)** $A = B = 0$

(D) $A = B \neq 0$ **(E)** $A \neq B$, and the acceleration magnitudes depend on the force the ball was kicked with



16. *(2 points)* A mechanical device made of ideal pulleys that can be considered small and massless, passed over by a light long enough for an inextensible thread and is mechanically loaded with two weights, A and B, held at rest far away from the ground. (See the figure.) If both air resistance and buoyancy, as well as all dissipative frictions, can safely be ignored, which one among the following statements most closely describes the motion of the two weights through the air upon their release?



(**A**) The motion of the two weights is under the sole influence of gravity so that they will undergo free fall.

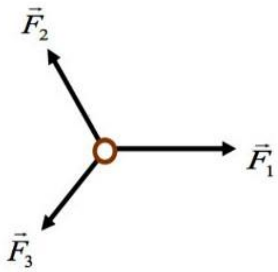
(**B**) Weight A will accelerate upwards with twice the acceleration value of weight B moving downwards.

(**C**) More information about the masses of the two weights is required.

(**D**) Weight B will accelerate upwards with twice the acceleration value of weight A moving downwards.

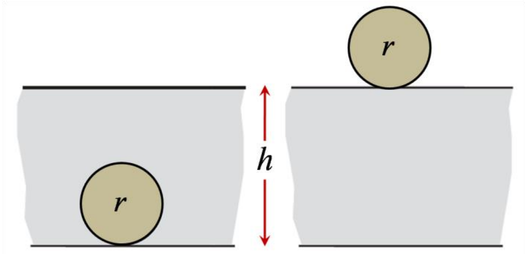
(**E**) The two weights will always hang motionless in the air.

17. *(2 points)* Three inextensible threads are tied to a tiny ring and pulled by hand along three arbitrary directions in the horizontal plane. See the pulling force configuration in the figure $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Which among the triples of the pulling force magnitudes (expressed in N units) will never balance out or result in the ring equilibrium?



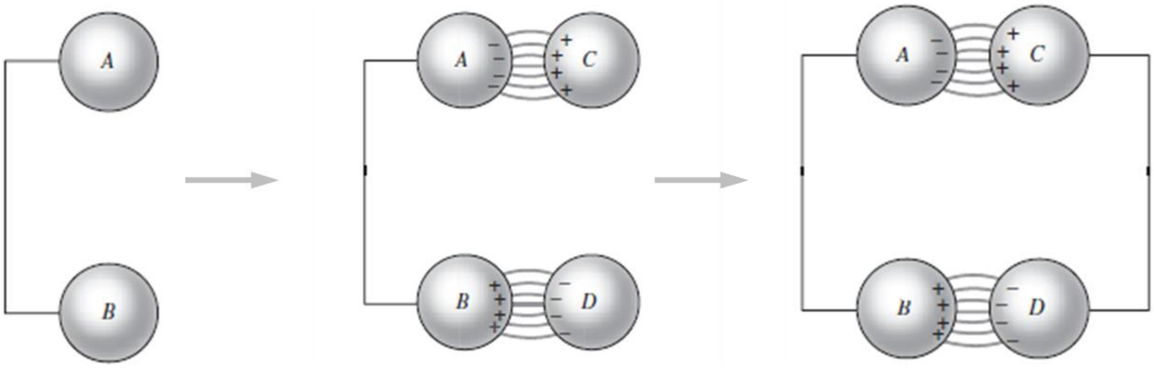
(**A**) $(3, 4, 5)$ (**B**) $(10, 5, 6)$ (**C**) $(8, 7, 6)$ (**D**) $(3, 7, 2)$ (**E**) $(12, 5, 13)$

18. *(2 points)* A solid ball of mass m and radius r rests at the bottom of a large water reservoir, as shown in the left figure. The depth of the reservoir is h. The mass density of the material of which the ball is made coincides with that of water. Now the ball is gradually pulled completely out of the water into the position as shown in the right figure. Which one of the following statements is the correct expression for the mechanical work done by the agent pulling the ball out? The gravity acceleration is g.



(**A**) $mg(h + r)$ (**B**) $mgr/2$ (**C**) mgh (**D**) $mg(h + r/2)$ (**E**) mgr

19. *(2 points)* The two metal spheres A and B are connected by a wire of zero total charge (left figure). Two other oppositely charged conducting spheres, C and D, have been brought into the positions shown, inducing



charges of opposite signs in A and in B (the figure in the middle). If now C and D are connected by a wire (right figure), it could be argued that something like the charge distribution ought to persist, each charge concentration being held in place by the attraction of the opposite charge nearby. Among the following statements describing the ultimate situation (right figure), find the false one.

(**A**) The sum of all the electric *potential* differences around the final loop (closed rectangular path) is not *zero.*

(**B**) The proposed situation can never represent an ultimate stationary charge distribution.

(**C**) The proposed situation suggests that a static electric field is capable of doing finite work along a closed path.

(**D**) The proposed charge distribution can be maintained for a long time if the wires are held by the external mechanical force to make a balance with the electrostatic attractive force originating from A and C, B and D.

(**E**) The whole system is to cascade down to zero charge everywhere as soon as C and D get wired.

20. *(2 points)* If the electric resistance of a small immersion heater designated for heating some water for tea or soup at home is increased (see the figure), what can you say about the rate of the heating process?

(**A**) it will slow down

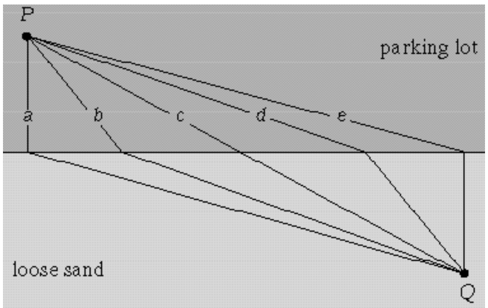
(**B**) it will initially slow down to ultimately speed up

(**C**) there is insufficient information

(**D**) it will speed up

(**E**) it will remain unchanged

21. *(2 points)* A runner wants to get from P to Q as fast as he can. He can run faster on the pavement (a parking lot) than on loose sand. Which path should he take (see figure)?

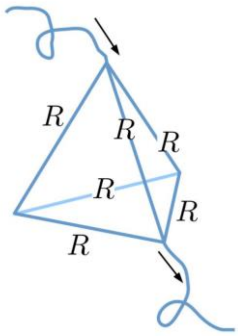


(**A**) a (**B**) b (**C**) c (**D**) d (**E**) e

22. *(3 points)* Imagine that the world globe (the entire planet Earth sphere) is placed on a giant circular plane mirror touching the center of the mirror with its South geographic pole. In terms of Earth's radius R, find the minimum radius of the mirror so that image of the latitude of $\phi = 30°$ N, passing just south of Cairo in Egypt, becomes visible in the mirror.

(**A**) $2R$ (**B**) $\sqrt{3}\,R$ (**C**) $3R/2$ (**D**) $3R$

(**E**) $\sqrt{3/2}\,R$



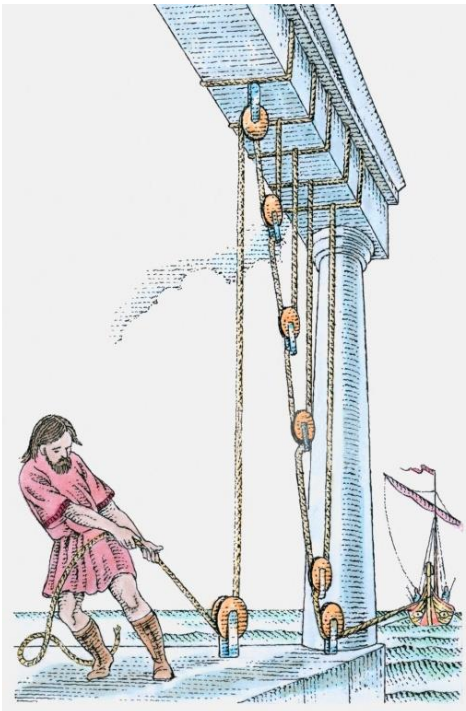
23. *(4 points)* The figure shows a network of identical resistors of resistance R in the shape of a tetrahedron. What is the equivalent resistance of such circuit?

(**A**) $2R$ (**B**) $R/2$ (**C**) $3R/2$ (**D**) $5R/2$ (**E**) $6R$

24. *(4 points)* Three different liquids A, B and C of equal masses have the following temperatures: 10℃, 16℃ and 28℃ respectively. If we mixed liquids A and B their equilibrium temperature would be 12℃. If we did the same with liquids B and C their equilibrium temperature would be 25℃. What would be the temperature of the liquids A and C mixture?

(**A**) 23,8℃ (**B**) 17,5℃ (**C**) 12,4℃ (**D**) 20,8℃ (**E**) 24,2℃

25. *(4 points)* The ancient Greeks were already familiar with the mechanical advantages of complex pulley systems. In order to haul a sailing ship which has run aground into open water, a Greek man employs a system of light pulleys with long resilient inextensible ropes, as is given in the figure. The ship needs to travel a 10 m distance to arrive at the dock, while a water-opposing force of about 3200 N must be exceeded in order for the ship to move. The force remains nearly constant during the ship's motion at a slow uniform speed. If A_{min} stands for the least amount of the mechanical work done by the man to dock the ship steadily and if F_{min} is the minimum value of his stationary pulling force applied to the rope, then:



(**A**) $A_{min} = 32$ kJ, $F_{min} = 200$ N

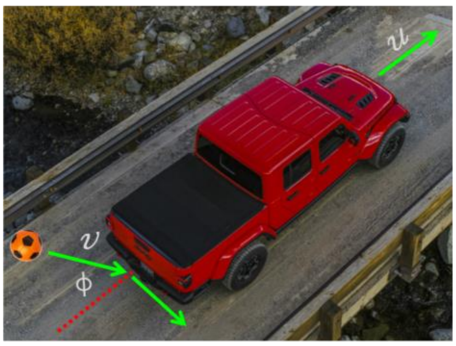
(**B**) $A_{min} = 2$ kJ, $F_{min} = 200$ N

(**C**) $A_{min} = 32$ kN, $F_{min} = 3200$ N

(**D**) $A_{min} = 2$ kN, $F_{min} = 3200$ N

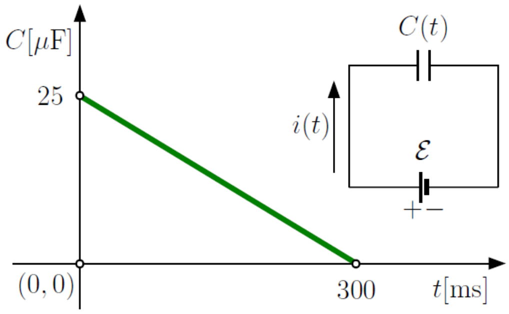
(**E**) $A_{min} = 8$ kN, $F_{min} = 800$ N

26. *(4 points)* A soccer ball flying at the speed of $v = 32$ m/s hits a vehicle in motion from behind, as shown in the figure. The vehicle moves at uniform speed u on a horizontal straight road. Spanning a horizontal plane, the angle between the vehicle's straight-line trajectory and the direction of the ball motion amounts to $\phi = 60°$. The ball bounces off the vertical rear-bumper wall perpendicularly to the direction of the vehicle motion so as to stay within the initial horizontal plane. Compute the value of vehicle velocity u. The ball is made of a perfectly elastic material. Ignore gravity, the spinning of the ball, and its dimensions.



(**A**) $u = 32$ m/s (**B**) $u = 16$ m/s (**C**) $u = 8\sqrt{3}$ m/s (**D**) $u = 8$ m/s (**E**) $u = 4\sqrt{3}$ m/s

27. *(2 points)* An ideal battery with $\mathcal{E} = 12\,V$ of its terminal voltage is loaded with a variable capacitor whose capacitance $C(t)$ is made drop linearly down to zero in time t, as shown in the figure. See both the C versus t diagram and the $\mathcal{E}$-$C(t)$ circuit inset. Observing the specified sense of the electric current flow, which one of the following statements for the value $i(t)$ of the electric current over the capacitance drop interval is correct?



(A) no current flows, $i(t) = 0$ **(B)** $i(t) = -1.0$ mA **(C)** $i(t)$ increases with time

(D) $i(t) = +1.0$ mA **(E)** $i(t)$ decreases with time

28. *(3 points)* The figure shows a converging thin lens held above three equal-sized letters A. To your left, the lens is 5 cm away from the paper, while to your right, the lens is 15 cm away from the paper. Estimate the focal length of lens f.

(A) $f \approx 30$ cm

(B) $f \approx 15$ cm

(C) $f \approx 20$ cm

(D) $f \approx 5$ cm

(E) $f \approx 10$ cm

29. *(6 points)* While rewinding a video cassette radius of the tape wound on the left reel R_1 increases while on the right the radius of the wound tape R_2 decreases. Which quantity relating R_1 to R_2 is constant?

(A) $R_1 R_2$

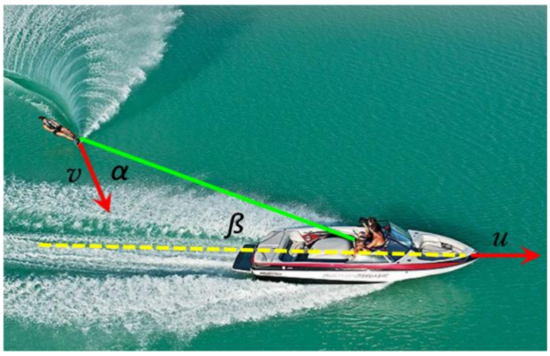
(B) $R_1 + R_2$

(C) $R_1^2 + R_2^2$

(D) $1/R_1 + 1/R_2$

(E) $1/R_1^2 + 1/R_2^2$

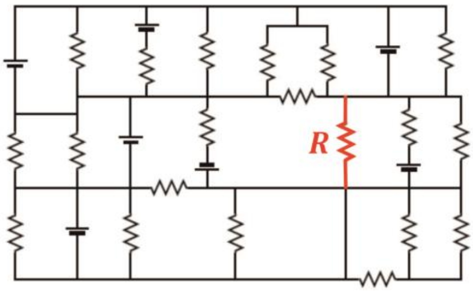
30. *(6 points)* A powerboat is moving with a uniform velocity u pulling behind a water-skier with the help of an inextensible tug-rope, as shown in the figure. To speed up, the water-skier tilts the skis slightly away from the rope direction by angle α. In this way, the rope remains tense, forming angle β with respect to the powerboat motion direction. Find the skier instantaneous velocity v relative to the still water if $\alpha = 45°$ and $\beta = 30°$.



(A) $v = \sqrt{2}\,u$ **(B)** $v = \sqrt{2/3}\,u$

(C) $v = \sqrt{3}\,u$ **(D)** $v = \sqrt{6}\,u$ **(E)** $v = \sqrt{3/2}\,u$

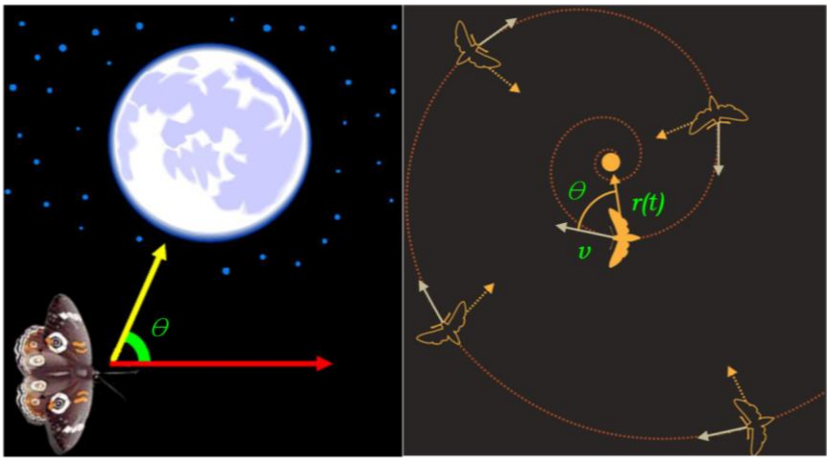
31. *(6 points)* The figure on the right shows an electric circuit containing identical resistors with 2Ω resistance and identical batteries with EMF of $6V$. Find the electric current going through resistor R.



(A) 0,5A **(B)** 18A **(C)** 9A

(D) 2A **(E)** 6A

32. *(6 points)* Nocturnal insects fly at night relying on the moonlight. They utilize the so-called transverse navigation mechanism whereby a navigation angle θ is kept fixed to the moonlight rays while flying. Since the moon is far away, the mechanism guarantees straight-line flights (left figure). However, light pollution decimates the nocturnal insects in populated areas. For instance, as soon as the attention of a moth is drawn by a nearby artificially made point-like light source, it keeps maintaining the navigation angle with respect to the radially emitted light rays. The moth eventually enters a deadly spiral flight-path in one plane (the right figure) flying with $\theta = 60^{\circ}$. The momentary distance between the light source and the moth, $r(t)$, decreases from its initial value R down to zero when the insect fatally ends up into the light source to scorch. How much does the total distance, s, travelled by the moth amount to?



(A) $s = 2R\sqrt{3}$ **(B)** $s = R/2$ **(C)** $s = 2R$ **(D)** $s = R\sqrt{3}/2$ **(E)** $s = R\sqrt{3}$

The end!