The Mathematical Grammar School Cup

Physics Competition Solutions

24.06.2021.

# of question	answer	# of question	answer
1.	**E**	17.	**D**
2.	**B**	18.	**E**
3.	**E**	19.	**D**
4.	**B**	20.	**A**
5.	**D**	21.	**D**
6.	**A**	22.	**B**
7.	**C**	23.	**B**
8.	**B**	24.	**D**
9.	**C**	25.	**A**
10.	**C**	26.	**D**
11.	**E**	27.	**B**
12.	**D**	28.	**E**
13.	**B**	29.	**C**
14.	**B**	30.	**E**
15.	**D**	31.	**E**
16.	**A**	32.	**C**

1. (**E**) George Ohm.

2. (**B**) V/A.

3. (**E**) Coulomb, Faraday, Maxwell.

4. (**B**) refraction.

5. (**D**) Ole Roemer.

6. (**A**) resonance.

7. (**C**) $\overline{AB} = 30$ km and $a_{max} \approx 1/27$ m/s^2. Using the symmetry property of $v(t)$, the area of the region bounded by the AB curve gives the distance $\overline{AB}$ as $6 \times \Delta s + 6 \times \Delta s + 6 \times \Delta s = 18 \times \Delta s$, where Δs stands for the area of the elementary square $5\,\text{min} \times 20\,\text{km} = 5/3\,\text{km}$. The tangent to $v(t)$ curve on the graph point $(120\text{ min, } 60\text{ km/h})$ has the largest slope of $\frac{\Delta v}{\Delta t} = \frac{2\times 4\frac{\text{km}}{\text{h}}}{1\text{ min}}$. This value nearly corresponds to the maximum acceleration value.

8. (**B**) $c = 5.00$ m/s. The two nearest neighbors stay far away at $l = \frac{L}{n-1} = 0.55$ m distance from each other. The information travels at the speed of $c = l/\tau$.

9. (**C**) purely kinetic.

10. (**C**) $Q_0 < Q$. Let V_0 respectively be the volume of metal ball at 0 °C and let V be the corresponding value at t. Let $M/V_0{}^*$ and $V_0{}^*$ be the mass density and volume of alcohol at 0 °C and let M/V^* and V^* respectively be the corresponding values at t. By increasing temperature, the two volumes evolve as: $V = V_0(1+\alpha t)$ and $V^* = V^*{}_0(1+\alpha^* t)$, where α and α^* are constants measuring the thermal expansion rates of metal ball and alcohol, respectively. It is sufficient to compare the buoyancy forces, F_0 at 0 °C and F at t. Namely, $\frac{F_0 - F}{g} = \frac{M}{V_0{}^*}V_0 - \frac{M}{V^*}V = \frac{MV_0}{V_0{}^*}\left(1 - \frac{1+\alpha t}{1+\alpha^* t}\right) = \frac{MV_0}{V_0{}^*}\frac{(\alpha^* - \alpha)t}{1+\alpha^* t} > 0$.

11. (**E**) ⊙⊙⊙⊗⊗.

12. (**D**) zero for L and clockwise for R. Apply the Lenz law.

13. (**B**) The ripples on the water surface result in random incident reflection angles falling on the observer's eye across the distance of the water's surface - an illusion of a continuously elongated image is created.

14. (**B**). Before the ground gets hit by the body B, $v_B(t) = gt, v_A(t) = const - gt$, so that the relative velocity remains constant. Afterwards, only the body A moves falling freely, while B stays stationary.

15. (**D**) $A = B \neq 0$. The motion is under the sole influence of the vertical gravity acceleration ($a = g = const \neq 0$).

16. (**A**) The motion of the two weights is under the sole influence of gravity so that they will undergo free fall. Use the fact that the tension force T in the thread is zero as $T = 2T$.

17. (**D**) $(3, 7, 2)$. It is impossible to construct a triangle with such side-lengths.

18. (**E**) mgr. As the water reservoir is large, its water level can be considered unaffected during the action of the agent. No mechanical work is done till the uppermost ball point is made surfaced. Then, the agent must do a mechanical work of $2mgr$, yet helped with the work of mgr as a result of discharging downwards the same amount of water from the large reservoir into the spherical void.

19. (**D**) The proposed charge distribution can be maintained for a long time if the wires are held by the external mechanical force to make a balance with the electrostatic attractive force originating from A and C, B and D.

20. (**A**) it will slow down. Use the fact that the electric power will decrease due to $P = U^2/R$. U is the standardized terminal voltage at home; the electric resistance R is increased.

21. (**D**) d. Apply an analogy to the refraction law from wave mechanics. The refraction angle comes nearer the normal line as compared to the incident angle.

22. (**B**) $\sqrt{3}R$. Construct the tangent to a point (C) belonging to the latitude circumference. Line OC spans $30°$ angle with the equatorial plane, or the mirror plane alternatively, where O is the Earth center. The minimum radius required is found as the segment bounded with South pole (S) and the intersection point (R) between the mirror and tangent. Next, use the property of deltoid $SRCO$.

23. (**B**) $R/2$. Use the symmetry property whereby there is no potential difference across the branch connected against the branch bounded with the incoming and outgoing current node.

24. (**D**) 20,8°C.

25. (**A**) $A_{min} = 32$ kJ, $F_{min} = 200$ N. By the energy conservation law, the mechanical work done by the Greek man is at least equal to the mechanical work done by the pulling force which acts on the ship to make it slowly arrive at the dock. On the other hand, the pulling force of the Greek man is 16 times weaker than $3200\ N$.

26. (**D**) $u = 8$ m/s. Within the vehicle frame, the soccer ball hits it at a relative speed of $v_{rel} = v/2 - u$, along the road direction. The ball rebound at the same speed in the opposite direction. Now, the ball velocity relative to the road amounts exactly $v_{rel} - u$. This value must be zero, so that $u = v/4$.

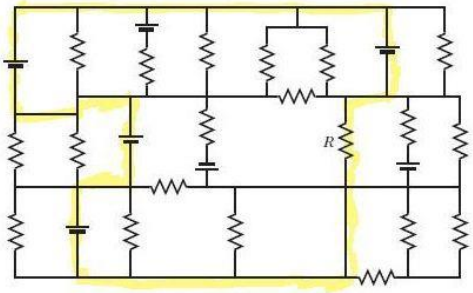
27. (**B**) $i(t) = -1.0$ mA. The charge at the capacitor will change according to $q(t) = \mathcal{E}C(t)$. This means that the current $i(t) = \frac{\Delta q}{\Delta t} = \frac{\Delta C}{\Delta t}\mathcal{E} = -1.0$ mA, implying that the capacitor is getting discharged. The negative slope, $\frac{\Delta C}{\Delta t} = -\frac{250\ \mu\text{F}}{300\ \text{ms}}$, can be read out from the graph.

28. (**E**) $f \approx 10$ cm. Magnifications in the two figures appear to have the same magnitude with the magnification positive in the left figure and negative in the right one. Using the magnification equation, we can write the image distance $l_2 \approx -\frac{p_2}{p_1}l_1 = -\frac{15\text{ cm}}{5\text{ cm}}l_1 > 0$ (right) in terms of the image distance $l_1 < 0$ (left). Finally, the focal distance f is calculated by using the thin lens equation for each image: $\frac{1}{f} = \frac{1}{p_1} + \frac{1}{l_1} = \frac{1}{p_2} + \frac{1}{l_2}$.

29. (**C**) $R_1^2 + R_2^2$. Use the fact that both tape thickness (d) and length (L) are invariant. As d is constant, the left {right} winding number reads as $N_1 \approx (R_1 - R_0)/d$ $\{N_2 \approx (R_2 - R_0)/d\}$ at any instance, where R_0 is the radius of the reel itself. On the other hand, $L = const$ leads to $N_1R_0 + \frac{N_1(N_1+1)d}{2} + N_2R_0 + \frac{N_2(N_2+1)d}{2} \approx const$. Use the fact that large $N_{1,2} + 1 \approx N_{1,2}$.

Another way to prove this would be to notice that the side area of the tape that we see is equal to the product $L \cdot d = const$ while on the other hand this area is equal to $(R_1^2 - R_0^2)\pi + (R_2^2 - R_0^2)\pi$. From there it follows that $R_1^2 + R_2^2 = const$.

30. (**E**) $v = \sqrt{3/2}\,u$. Velocity components along the inextensible rope must be equal, that is, $v\sqrt{2}/2 = u\sqrt{3}/2$.



31. (**E**) 6 A. Use a loop with containing only batteries and that particular resistor R, as highlighted in the figure for example.

32. (**C**) $s = 2R$. At any point of the insect motion, the length of the substantially small path travelled by the insect, Δs, is twice the ΔR value, where ΔR represents the substantially small radial displacement of the insect towards the point-like light source. Ultimately, the total radial displacement of the insect will coincide with the initial distance R.