5^{th} Math Cup - Physics Competition Solutions

(1) a. Answer: (C). $E = P \cdot t_{average} \cdot N_{days} = 200W \cdot 6 \cdot 60 \cdot 60s/day \cdot 30days = 129.600.000J = 129.6MJ$.
b. Answer: (C). $W = \frac{J}{s} = \frac{N \cdot m}{s} = \frac{\frac{kgm}{s^2} \cdot m}{s} = \frac{kgm^2}{s^3}$.

(2) Answer: (C). $p = \frac{mg}{4S}$ which gives $m = \frac{4pS}{g} = \frac{4 \cdot 187500Pa \cdot 0.22m^2}{10N/kg} = 16500kg = 16.5t$.

(3) Answer: (A). Only concave mirrors can be used for magnification.

(4) Answer: (C). We first calculate the heat: $Q = mc\Delta T = \rho V c \Delta T = 1000kg/m^3 \cdot 0.01m^3 \cdot 4200J/kg°C \cdot 30°C = 1.260.000J$. On the other hand, $1kWh$ of electrical energy equals: $1000W \cdot 3600s = 3.600.000J$. Using simple proportion, we get that single dish wash costs: $1.26 \cdot 8$ dinars$/3.6 = 2.8$ dinars.

(5) Answer: (C). The boy sees vertical trajectory, onlooker standing by the side of the road sees parabolic trajectory due to relative motion. When the car decelerates, the ball will fall in front of boy's hand.

(6) Answer: (C). The car is first decelerating, then accelerating and all the time coordinate x is increasing.

(7) Answer: It decreases. When we insert a piece of demagnetized iron in the coils, static magnetic field will increase. Moreover, magnetic flux also increases and by Lenz rule, induced current will have the opposite direction.

(8) a. Answer: It doesn't change. Cords are made of insulating material.
b. Answer: It increases. The balls are identical and made of conducting material, which means that charge will be equally distributed between them after their contact. Charge of each ball will be $Q = \frac{q_1+q_2}{2}$. Coulomb force is proportional to the product of the charges involved. That product takes maximal value when the charges are equal, which can easily be verified using an inequality $q_1 q_2 \leq (\frac{q_1+q_2}{2})^2$. This means that the angle becomes greater than in previous equilibrium state.

(9) a. Answer: It doesn't change. The image will be formed in the focal plane of the bottom lens.
b. Answer: It doesn't change. Magnification of the system of theses lenses doesn't depend on the position of the bottom lens. Size of the image equals the size of the object.
c. Answer: It increases. The image is formed in focal plane of the bottom lens. Due to the change of the refractive index, the focal length of the bottom lens will increase.

(10) Answer: $A > B$. At the moment of release, acceleration of the lower ball equals zero (gravitational and elastic force cancel each other), while acceleration of the upper ball equals $2g$.

(11) Answer: A=B. Hydrostatic pressure is the same at the bottom of the containers.

(12) We can solve this problem using parallel connection of two batteries, each containing $n = 350.000$ electro plaques connected in series. Current flowing through water is given by: $I = \frac{nE_e}{\frac{nr_e}{2} + R_v} \approx 220mA$, which is more than enough to paralyze human muscles.

(13) Dependence of distance between airplanes on time is given by: $d^2 = 2a^2 + (v_1^2 + v_2^2)t^2 - 2a(v_1 + v_2)t = (1000\frac{km}{h}t - 28km)^2 + 16km^2$, from which we get $d_{min} = 4km$. The problem can also be solved using relative velocity and similar triangles.

(14) The droplet will accelerate until the equilibrium of gravitational pull and resistive force. Maximum speed of the droplet is $v = mg/k = \rho V g/k$. Kinetic energy at the moment of collision is $E_k = \frac{mv^2}{2} = \frac{\rho^3 V^3 g^2}{2k^2}$ and 40% of that energy is converted into the heat. Ratio of the two heats equals: $\frac{Q_{Hg}}{Q_{H_2O}} = (\frac{\rho_{Hg}}{\rho_{H_2O}})^3 \approx 2515$. Here we neglected the buoyancy force.

(15) Slowly tilt the vessel until the free surface of the water touches the edge of both upper and lower bases of the vessel.

(16) In both cases it is important to close the electric circuit. The electric circuit powered by the tram motor closes through the metal rails while the trolley is isolated from the ground by rubber tires, so the second conductor is required.

(17) The density of air, and therefore the index of refraction, depends on temperature. Warm air has a smaller refractive index. Above the hot sand or above the grill fire, the air is very warm. Such warm air moves upwards, because it is less dense, and the colder falls and receives heat from sand or fire, and it becomes hotter and climbs up again. This movement of air causes changes in the physical characteristics of the environment. Such an environment seems fluid (not in terms of aggregate state, but with the variability of physical properties), so the objects is blurry in the eyes of an observer. Refraction of light is time dependent, and therefore objects do not have a constant form for observers. On the other hand, due to the variable refractive index, which depends on height (in the lowest layers is the warmest air with the smallest index of refraction, and above the air is colder with a larger refractive index), so the light rays do not propagate along a straight trajectory. It may happen that the rays coming from far away look like they are coming from lower layers of the air. That is why what we see in such a situation is not real, it is just an illusion.